Calcola le seguenti derivate con logaritmi ed esponenziali

1. $\boldsymbol{y = 3^x + 2lnx}$

$$y' = D(y) = D(3^x) + 2D(lnx) = 3^x ln3 + \frac{2}{x}$$

2. $\boldsymbol{y = 2e^x - 3lnx - 5x}$

$$y' = D(y) = 2D(e^x) - 3D(lnx) - 5D(x) = 2e^x - \frac{3}{x} - 5$$

3. $\boldsymbol{y = e^x - 2lnx + 3}$

$$y' = D(y) = D(e^x) - 2D(lnx) + D(3) = e^x - \frac{2}{x}$$

4. $\boldsymbol{y = e^{x-3} = \frac{e^x}{e^3}}$

$$y' = D(y) = \frac{1}{e^3}D(e^x) = \frac{e^x}{e^3} = e^{x-3} = y$$

5. $\boldsymbol{y = 2cosx - \frac{1}{3}lnx + x^2}$

$$y' = D(y) = 2D(cosx) - \frac{1}{3}D(lnx) + D(x^2) = -2sinx - \frac{1}{3x} + 2x$$

6. $\boldsymbol{y = 2a^x + 5}$

$$y' = D(y) = 2D(a^x) + D(5) = 2a^x lna$$

7. $\boldsymbol{y = 2lnx - 3y = \frac{1-3senx}{x^2}}$

$$y' = D(y) = \left(\frac{D(1-3senx)x^2 - (1-3senx)D(x^2)}{(x^2)^2}\right) = \frac{-3x^2cosx - (1-3senx)(2x)}{x^4}$$
$$= \frac{-3x^2cosx - 2x + 6xsenx}{x^4} = \frac{x(-3xcosx - 2 + 6senx)}{x^4} = \frac{-3xcosx - 2 + 6senx}{x^3}$$

8. $\boldsymbol{y = \frac{x+cosx}{senx}}$

$$y' = D(y) = \frac{D(x+cosx)\cdot senx - (x+cosx)\cdot D(senx)}{sen^2x} = \frac{(1-senx)senx - cosx(x+cosx)}{sen^2x}$$
$$= \frac{senx - sen^2x - xcosx - \cos^2 x}{sen^2x} = \frac{senx - xcosx - (sen^2x + \cos^2 x)}{sen^2x}$$
$$= \frac{senx - xcosx - 1}{sen^2x}$$

9. $\boldsymbol{y = x^x}$

$$lny = \ln(x^x) \rightarrow lny = xlnx$$

$$D(lny) = D(xlnx)$$

$$\frac{1}{y}y' = xD(lnx) + lnxD(x)$$

$$\frac{1}{y}y' = \frac{x}{x} + lnx$$

$$\frac{1}{y}y' = 1 + lnx$$

$$y' = y(1 + lnx) = x^x(1 + lnx)$$

10. $\boldsymbol{y = (senx)^{lnx}}$

$$lny = \ln\left(senx^{lnx}\right) \rightarrow lny = lnx\ ln(senx)$$

$$D(lny) = D[lnx \cdot \ln(senx)]$$

$$\frac{1}{y}y' = D(lnx) \cdot \ln(senx) + D[\ln(senx)] \cdot lnx$$

$$\frac{1}{y}y' = \frac{\ln(senx)}{x} + lnx \cdot \frac{cosx}{senx}$$

$$\frac{1}{y}y' = \frac{\ln(senx)}{x} + lnx \cdot cotgx$$

$$y' = y\left[\frac{\ln(senx)}{x} + lnx \cdot cotgx\right] = (senx)^{lnx}\left[\frac{\ln(senx)}{x} + lnx \cdot cotgx\right]$$